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# Modified response spectrum approach for multiply-supported secondary systems

A. Saudy[I], A. Ghobarah[II], and T.S. Aziz[II]

## ABSTRACT

An alternative technique has been developed to evaluate the ordinates of the Cross Cross Floor Spectra (CCFS). The technique properly accounts for the dynamic interaction, tuning and non-classical damping characters of the combined Primary-Secondary (P-S) systems. The approach can estimate the peak response of the tuned, non-classically damped P-S systems more accurately. In the analysis, two fictitious oscillators are attached to the primary system in the course of evaluating the ordinates rather than attaching only one oscillator to the primary system as was previously suggested. A model for the combined P-S systems is analyzed by the original and the proposed techniques. The results are compared with the response values obtained using coupled dynamic analysis. The proposed technique proved to be more accurate in estimating the peak response of the secondary system, specially in cases of tuned, non-classically damped P-S systems.

## INTRODUCTION

In industrial facilities and nuclear power plants, relatively light structures are normally attached to heavier ones. Normally, the lighter structures are considered as secondary systems to the supporting structures which are the primary systems. The secondary systems are generally attached to the primary ones at several attachment points. The seismic behaviour of multiply supported Multi-Degree-Of-Freedom (MDOF) secondary systems has received considerable attention due to the vital role such systems play in regard to safety. To investigate the seismic behaviour of multiply supported secondary systems two aspects need to be addressed. Firstly, the dynamic characteristics of the combined primary-secondary (P-S) system have to be accounted for. These characteristics include : dynamic interaction, tuning, non-classical damping and spatial coupling. Secondly, in addition to the different support accelerations, the attachment points normally undergo differential support motions. This, in turn, will lead to increased stresses in the secondary system.

In general, the different approaches adopted in the seismic analysis of MDOF multiply supported secondary systems can be classified as coupled and uncoupled analyses. In theory, the exact response of a general secondary system can be obtained by using standard methods of coupled dynamic analysis for the combined P-S system. Due to many practical difficulties in carrying out a coupled dynamic analysis,

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[I]Graduate student, Department of Civil Engineering and Engineering Mechanics, McMaster University, Hamilton, Ontario.

[II]Professor, Department of Civil Engineering and Engineering Mechanics, McMaster University, Hamilton, Ontario.

an uncoupled approach is traditionally adopted, (Amin et al, 1971; Lee et al, 1983; Shaw, 1975; Vashi, 1975). In the uncoupled analysis, complex structures are commonly subdivided into two subsystems; primary and secondary; then, analyzed separately. Such analysis completely ignores the dynamic characteristics of the combined P-S system. Moreover, the seismic responses of the secondary system are partitioned into two components; dynamic and pseudo-static.

Recently, the Cross Cross Floor Spectrum (CCFS) technique has been developed based on the principles of random vibration and stochastic analysis, (Asfura et al, 1986). Although the technique is based on an uncoupled analysis approach, it attempts to account for some of the dynamic characteristics of the combined P-S system. This approach proved to be inaccurate in predicting the response of tuned, non-classically damped combined P-S systems.

The objectives of this study are to manifest the sources of error in the original CCFS approach and to propose a modified CCFS technique that properly accounts for the dynamic interaction and tuning.

## STATEMENT OF THE PROBLEM

Consider the model for the combined P-S system subjected to base excitation $u_g(t)$ as shown in Fig. 1. It is assumed that both the primary and secondary systems are linear elastic, viscously and classically damped. The secondary system is attached to the primary system at various points. The primary system would respond to the base excitation differently at the attachment points. Accordingly, the secondary system will be subjected to different or multiple accelerations. These accelerations are normally different in both phase and amplitude. In addition, the attachment points will exhibit differential movements which would cause stresses in the secondary systems.

Based on the principles of random vibration and stochastic analysis, the CCFS approach has been developed by Asfura and Der Kiuregian (1986). In their work, they employed the idea of attaching two fictitious oscillators that have two frequencies of those of the secondary system; i.e. $\omega_i$ and $\omega_j$, at two support points (floors) of the primary system; i.e. K and L, Fig. 2. Accordingly, modal combination rules were suggested to predict responses at the $r^{th}$ degree of freedom of the secondary system independently on the primary system. These rules lead to the mean of the peak acceleration at the $r^{th}$ degree of freedom of the secondary system as;

$$E[\ddot{u}_{r,max}] = [\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}\, a_{rj} \sum_{K=1}^{n_s}\sum_{L=1}^{n_s} b_{iK}\, b_{iL}\, S_{KL}^{a}(\omega_i, \xi_i; \omega_j, \xi_j)]^{1/2} \tag{1}$$

In a similar manner, the mean of the peak relative displacement of the $r^{th}$ degree of freedom is given by

$$E[v_{r,max}] = [\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}\, a_{rj} \sum_{K=1}^{n_s}\sum_{L=1}^{n_s} b_{iK}\, b_{iL}\, S_{KL}^{v}(\omega_i, \xi_i; \omega_j, \xi_j)]^{1/2} \tag{2}$$

where

$$a_{ri} = \frac{\phi_{ri}}{m_i\, \omega_i^2} \tag{3}$$

and

$$b_{iK} = \sum_{m=1}^{n} \phi_{mi}\, k_{c_{mK}} \tag{4}$$

$S_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)$ is the ordinate of a "cross-oscillator cross-floor" response spectrum associated with the motions of the $K^{th}$ and $L^{th}$ floors of the primary system, Fig. 3. In other words, it is the mean of the peak response associated with the covariance of the responses of the two oscillators. $\phi_i$, $\omega_i$, $\xi_i$ and $m_i$ are the mode shape, modal circular frequency, modal damping factor and modal mass associated with the $i^{th}$ mode of vibration of the secondary system. $K_c$ is the conventional coupling stiffness matrix between the two subsystems.

Theoretically, the ordinates of the CCFS are evaluated directly in terms of

- the input ground response spectrum, and
- the modal properties of the primary system.

The problem could be stated here as : how to evaluate such ordinates so that the dynamic characteristics of the combined P-S system could be properly accounted for?

In the original approach, a technique was suggested to evaluate the ordinates of the CCFS which account for the interaction and tuning effects. A mass value has been assigned to each oscillator. The mass value is calculated to bring about a shift in the nearest frequency of the primary system similar to that which actually takes place in the combined P-S system. Thus, according to a formula which is based on a tuning criterion, (Igusa et al, 1983), the mass values for the different oscillators are determined depending on the attachment point of each oscillator. Suppose that "N" is the number of degrees of freedom of the primary system, "n" is the number of degrees of freedom of the secondary system and "$n_a$" is the number of the attachment points supporting the secondary system. Thus, in order to evaluate a CCFS ordinate, an (N+2) DOF system, as that shown in Fig. 2, is studied. The (N+2) DOF system was replaced in the original approach by two (N+1) DOF systems, Fig. 4. Accordingly, ($n \times n_a$) different systems are analyzed. Each of these systems consists of the original primary system and an oscillator representing one of the secondary system modes. Thus, each system is an (N+1) DOF system. The effect of the non-classical damping character has been approximately accounted for based on another tuning formula, (Igusa et al, 1983). Finally, a modal combination rule for evaluating the CCFS ordinates $S_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)$ is developed. In this rule, a correlation coefficient, (Der Kiureghian, 1980), that accounts for the cross modal correlation between the two (N+1) DOF systems is employed.

It was observed that the CCFS approach gives accurate results in case of detuned secondary systems whether the damping is classical or non-classical. In case of tuned secondary systems, the CCFS approach overestimates the response. The error is greatly increased in the tuned, non-classical cases, (Asfura et al, 1986). A more accurate technique has to be developed in order to account properly for the compound effect of tuning and non-classical damping.

## ALTERNATIVE TECHNIQUE FOR EVALUATING CCFS ORDINATES

It is believed that the major sources of error arising in case of adopting the original technique to analyze tuned P-S systems could be attributed to the negligence of the great dynamic interaction in case of tuning. Although, the interaction effect is approximated by assigning mass values to the oscillator in each (N+1) DOF system, it is believed that, still in cases of tuned P-S systems, that effect is not considered properly. The alternative technique presented here is based on the fact that the (N+2) DOF system that has two oscillators with equal frequency can not be replaced with two similar (N+1) DOF systems. It is clear that the dynamic interaction between the two oscillators themselves is completely neglected if the technique of the (N+1) DOF system is followed. Moreover, the multiple tuning situation which arises due to coincidence of frequencies of the two oscillators and one (or more) of the frequencies of the primary system has also been ignored. To account for those neglected effects, the original (N+2) DOF system rather than the two (N+1) DOF systems has to be adopted in evaluating the CCFS ordinates.

To account for both the interaction and tuning effects, the idea of assigning equivalent mass values to the oscillators is again adopted. For the case of two oscillator with detuned frequencies, a mass value is assigned to each oscillator. Each mass value is equal to the that of a corresponding oscillator in an (N+1) DOF system. It has to be mentioned that this (N+1) DOF system is composed of the primary system to which is attached the oscillator at the same floor level as that in the analyzed (N+2) DOF system. For the case of two oscillators with tuned frequencies, each mass value is related to that of the corresponding (N+1) DOF system with a reduction factor ($\alpha$). The factor ($\alpha$) is introduced to account for the tuning effect between the two oscillators and the multiple tuning between the two oscillators and the primary system in case of tuned P-S systems. Several cases were analyzed to quantify the reduction factor ($\alpha$). It was found that a value that ranges between 0.9 and 1.0 is suitable for the cases of tuned P-S combined systems. For the cases of detuned P-S systems, a value of 0.75 could be used.

Accordingly, the proposed technique for evaluating the CCFS ordinates can be summarized in the following two steps:

1 - Determination of the dynamic modal properties of the (N+2) DOF systems. The modal frequencies and mode shapes are determined through direct analysis rather than employing perturbation techniques in order to achieve better accuracy. A reduction factor ($\alpha$) is applied to the equivalent masses assigned to the two oscillators when they have equal frequencies.

2 - Determination of the cross cross floor spectrum (CCFS) ordinates utilizing a modal combination rule to combine the modal responses of the $(N+1)^{th}$ and $(N+2)^{th}$ degrees of freedom in each (N+2) DOF system.

## NUMERICAL EXAMPLE

A model for the combined P-S systems is selected for analysis in order to examine the validity of the proposed (N+2) technique for evaluating the ordinates of the CCFS. The same model was analyzed before by Asfura and Der Kiureghian, (1986) subjected to an idealized ground response spectrum defined as

$$S_a(\omega,\xi) = g\left[\frac{\pi\omega}{2000\xi}\right]^{1/2} \quad (5)$$

The idealized acceleration ground response spectrum and the N-S component of ElCentro earthquake are employed as seismic inputs.

For comparison purposes, the theoretically "exact" responses have been obtained as well as the responses determined following the original (N+1) technique. The "exact" responses are determined by a coupled analysis of the combined P-S system.

A schematic representation of the model is shown in Fig. 5. The properties of the primary system are given in the same figure. Four cases are considered to account for both effects of tuning and non-classical damping. These cases are detuned, classically damped; detuned, non-classically damped; tuned, classically damped and tuned non-classically damped. Two different sets of mass and stiffness ratios were adopted to achieve the tuned and detuned cases. The sets of mass and stiffness ratios, the frequencies of the primary system and the modal damping factors for the two subsystems are tabulated in Table 1 for the all four cases.

Tables 2 and 3 summarize the estimated peak acceleration and displacement responses at the nodes of the secondary system subjected to the idelized ground response spectrum and to ElCentro earthquake. The coupled analysis as well as the two CCFS techniques are employed to determine the peak response of the model. The percentages of error in estimating the peak responses by the (N+1) and (N+2) technique are also tabulated. For the detuned cases, the reduction factor ($\alpha$) is assumed 0.75 while for the tuned cases, ($\alpha$) is assigned to 0.9 and 1.0 for classical and non-classical damping respectively.

It can be observed that the percentages of error in estimating the peak responses of the secondary system following the (N+1) technique is large, specially for the tuned, non-classically damped cases. The error percentage reaches 12% in the analyzed model when subjected to the idealized ground response spectrum. The error percentage exceeds 50% when subjected to ElCentro earthquake. It is also noticed that a more accurate response could be achieved by following the proposed (N+2) technique. The error percentages drops to less than 5% when the model is analyzed under the idealized ground response spectrum. When the model is analyzed under the ground acceleration history of ElCentro earthquake, the error percentages drops to about 20%. Employing the reduction factor ($\alpha$) in the tuned, non-classically damped cases greatly improves the predicted peak responses. In other words, using the (N+2) technique implies that more refined ordinates of the CCFS could be developed. In the mean time, the detuned cases indicate that a reduction factor ($\alpha$) of approximately 0.75 is essential to get accurate predictions of the peak response by the (N+2) technique. The percentages of error in those cases are comparable to those in the corresponding cases analyzed by the (N+1) technique.

## CONCLUSIONS

A modified CCFS approach that accounts for the dynamic interaction, tuning and non-classical damping was presented. An alternative technique for evaluating the ordinates of the cross cross floor spectra has been developed. While the original technique is based on analyzing a number of (N+1) DOF systems, the proposed technique is based on the analysis of a number of (N+2) DOF systems. Neglecting the tuning between the two oscillators themselves and the multiple tuning situation between the two oscillators and the primary system were found to be the major sources of error in estimating the response of tuned, non-classically damped combined P-S systems. The estimated peak responses of the tuned, non-classically P-S systems have been greatly improved by using a reduction factor ($\alpha$) applied to the equivalent masses assigned to the two oscillators when they have equal frequencies. The reduction factor ($\alpha$) ranges between 0.9 and 1.0 for the tuned P-S systems. For the detuned systems, the reduction factor ($\alpha$) may be assigned a 0.75 value. The percentages of error in case of tuned P-S systems significantly drop as a result of using this modified CCFS. This behavior is true whether the seismic input is in the form of a ground response spectrum or a ground time history.

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Table 1. Properties of the studied model

| | Primary System Properties | Secondary System Properties | | | |
|---|---|---|---|---|---|
| | | Detuned Classical Damped | Detuned Non-Classical Damped | Tuned Classical Damped | Tuned Non-Classical Damped |
| Frequencies in (rad/s) | 4.025 | 16.106 | 16.106 | 4.025 | 4.025 |
| | 11.750 | 22.361 | 22.361 | 5.588 | 5.588 |
| | 18.522 | 33.993 | 33.993 | 8.494 | 8.494 |
| | 23.794 | 38.730 | 38.730 | 9.678 | 9.678 |
| | 27.139 | 45.662 | 45.662 | 11.410 | 11.410 |
| Modal Damping Factor | 0.05 | 0.05 | 0.02 | 0.05 | 0.02 |
| m/M | ----- | 0.02 | 0.02 | 0.03203 | 0.03203 |
| k/K | ----- | 0.05 | 0.05 | 0.005 | 0.005 |

Table 2. Estimated peak response
(subjected to the idealized ground response spectrum)

| | DOF | Coupled | CCFS | | | | Coupled | CCFS | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | (N+1) | | (N+2) | | | (N+1) | | (N+2) | |
| | | Acc. (g) | Acc. (g) | %Error | Acc. (g) | %Error | Dis. (cm) | Dis. (cm) | %Error | Dis. (cm) | %Error |
| Detuned Classical Damped | 1 | 0.4784 | 0.467 | -2.38 | 0.475 | -0.71 | 25.60 | 25.60 | 0.0 | 25.70 | -0.39 |
| | 2 | 0.4572 | 0.442 | -3.32 | 0.456 | -0.26 | 23.45 | 23.45 | 0.0 | 23.54 | -0.39 |
| | 3 | 0.3956 | 0.398 | 0.61 | 0.398 | 0.61 | 20.50 | 20.50 | 0.0 | 20.50 | 0.00 |
| | 4 | 0.4065 | 0.401 | -1.35 | 0.406 | -0.12 | 16.87 | 16.78 | -0.58 | 16.87 | 0.00 |
| | 5 | 0.3606 | 0.354 | -1.83 | 0.359 | -0.44 | 12.65 | 12.46 | -1.55 | 12.56 | -0.72 |
| Detuned Non-Classical Damped | 1 | 0.5224 | 0.505 | -3.33 | 0.519 | -0.65 | 25.70 | 25.60 | -0.38 | 25.70 | 0.00 |
| | 2 | 0.5061 | 0.485 | -4.17 | 0.509 | 0.57 | 23.54 | 23.45 | -0.42 | 23.54 | 0.00 |
| | 3 | 0.4079 | 0.409 | 0.27 | 0.411 | 0.76 | 20.70 | 20.50 | -0.95 | 20.50 | -0.97 |
| | 4 | 0.4588 | 0.448 | -2.35 | 0.463 | 0.92 | 16.97 | 16.78 | -1.16 | 16.68 | -1.71 |
| | 5 | 0.4141 | 0.400 | -3.40 | 0.410 | -0.99 | 12.75 | 12.56 | -1.54 | 12.46 | -2.28 |
| Tuned Classical Damped | 1 | 1.0093 | 1.070 | 6.01 | 1.010 | 0.07 | 67.98 | 69.85 | 2.74 | 67.79 | -0.28 |
| | 2 | 1.3863 | 1.460 | 5.32 | 1.360 | -1.90 | 91.63 | 95.16 | 3.85 | 89.96 | -1.82 |
| | 3 | 1.1043 | 1.150 | 4.14 | 1.080 | -2.20 | 73.48 | 75.73 | 3.07 | 71.81 | -2.27 |
| | 4 | 1.3550 | 1.410 | 4.06 | 1.320 | -2.58 | 86.62 | 90.45 | 4.42 | 84.37 | -2.60 |
| | 5 | 0.9365 | 0.967 | 3.26 | 0.908 | -3.04 | 58.96 | 61.12 | 3.66 | 57.29 | -2.83 |
| Tuned Non-Classical Damped | 1 | 1.2844 | 1.420 | 10.56 | 1.290 | 0.44 | 84.07 | 90.35 | 7.47 | 85.45 | 1.64 |
| | 2 | 1.7790 | 1.990 | 11.86 | 1.760 | -1.07 | 114.68 | 126.55 | 10.35 | 115.76 | 0.94 |
| | 3 | 1.4181 | 1.570 | 10.71 | 1.400 | -1.28 | 92.12 | 101.04 | 9.69 | 92.31 | 0.21 |
| | 4 | 1.7688 | 1.950 | 10.24 | 1.740 | -1.63 | 110.17 | 122.63 | 11.31 | 109.87 | -0.27 |
| | 5 | 1.2325 | 1.340 | 8.72 | 1.210 | -1.83 | 75.24 | 83.29 | 10.69 | 74.56 | -0.90 |

Table 3. Estimated peak response
(subjected to ElCentro earthquake)

| | DOF | Coupled | CCFS | | | | Coupled | CCFS | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | (N+1) | | (N+2) | | | (N+1) | | (N+2) | |
| | | Acc. (g) | Acc. (g) | %Error | Acc. (g) | %Error | Dis. (cm) | Dis. (cm) | %Error | Dis. (cm) | %Error |
| Detuned Classical Damped | 1 | 0.2592 | 0.246 | -5.09 | 0.253 | -2.39 | 11.8716 | 11.8701 | -0.01 | 11.8701 | -0.01 |
| | 2 | 0.2217 | 0.211 | -4.83 | 0.218 | -1.67 | 10.8482 | 10.8891 | 0.38 | 10.8891 | 0.38 |
| | 3 | 0.2149 | 0.221 | 2.84 | 0.218 | 1.44 | 9.5422 | 9.5648 | 0.24 | 9.5648 | 0.24 |
| | 4 | 0.2675 | 0.267 | -0.19 | 0.271 | 1.31 | 7.9186 | 7.9460 | 0.35 | 7.9559 | 0.47 |
| | 5 | 0.2685 | 0.265 | -1.30 | 0.270 | 0.56 | 6.0404 | 6.0430 | 0.04 | 6.0528 | 0.21 |
| Detuned Non-Classical Damped | 1 | 0.2821 | 0.261 | -7.48 | 0.273 | -3.23 | 11.8742 | 11.8701 | -0.03 | 11.8701 | -0.03 |
| | 2 | 0.2473 | 0.229 | -7.40 | 0.245 | -0.93 | 10.8509 | 10.8891 | 0.35 | 10.8891 | 0.35 |
| | 3 | 0.2184 | 0.224 | 2.56 | 0.224 | 2.56 | 9.5425 | 9.5648 | 0.23 | 9.5648 | 0.23 |
| | 4 | 0.2874 | 0.280 | -2.57 | 0.291 | 1.25 | 7.9210 | 7.9461 | 0.32 | 7.9657 | 0.56 |
| | 5 | 0.2873 | 0.276 | -3.93 | 0.285 | -0.80 | 6.0428 | 6.0430 | 0.00 | 6.0528 | 0.17 |
| Tuned Classical Damped | 1 | 0.3825 | 0.509 | 33.07 | 0.420 | 9.80 | 20.8457 | 31.0977 | 49.18 | 26.5851 | 27.53 |
| | 2 | 0.4854 | 0.693 | 42.77 | 0.560 | 15.37 | 26.1432 | 42.8697 | 63.98 | 35.2179 | 34.71 |
| | 3 | 0.3782 | 0.548 | 44.90 | 0.446 | 17.93 | 20.4017 | 34.0407 | 66.85 | 28.1547 | 38.00 |
| | 4 | 0.4901 | 0.680 | 38.75 | 0.565 | 15.28 | 25.0189 | 41.3001 | 65.08 | 33.8445 | 35.28 |
| | 5 | 0.3529 | 0.475 | 34.60 | 0.402 | 13.91 | 17.1287 | 27.9585 | 63.23 | 23.0535 | 34.59 |
| Tuned Non-Classical Damped | 1 | 0.4841 | 0.598 | 23.53 | 0.473 | -2.29 | 25.2169 | 35.0217 | 38.88 | 28.8414 | 14.37 |
| | 2 | 0.6870 | 0.833 | 21.25 | 0.665 | -3.20 | 35.6884 | 49.6386 | 39.09 | 40.1229 | 12.43 |
| | 3 | 0.5648 | 0.670 | 18.63 | 0.546 | -3.33 | 28.8765 | 39.5343 | 36.91 | 32.3730 | 12.11 |
| | 4 | 0.7464 | 0.836 | 12.00 | 0.707 | -5.28 | 37.8870 | 48.9519 | 29.21 | 40.3191 | 6.42 |
| | 5 | 0.5313 | 0.586 | 10.30 | 0.506 | -4.76 | 26.0791 | 33.2559 | 27.52 | 27.6642 | 6.08 |

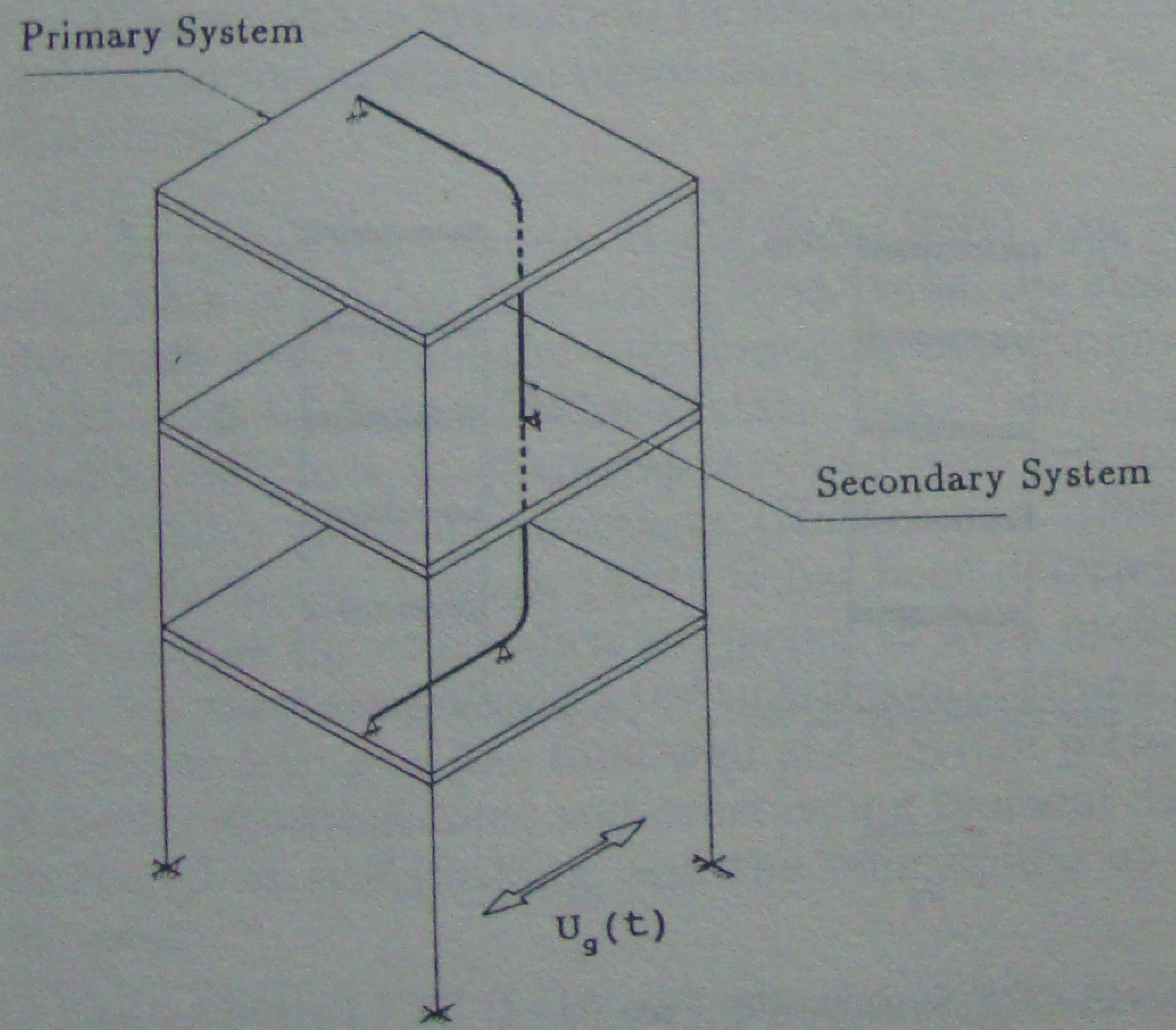


Figure 1. Combined Primary-Secondary (P-S) Systems

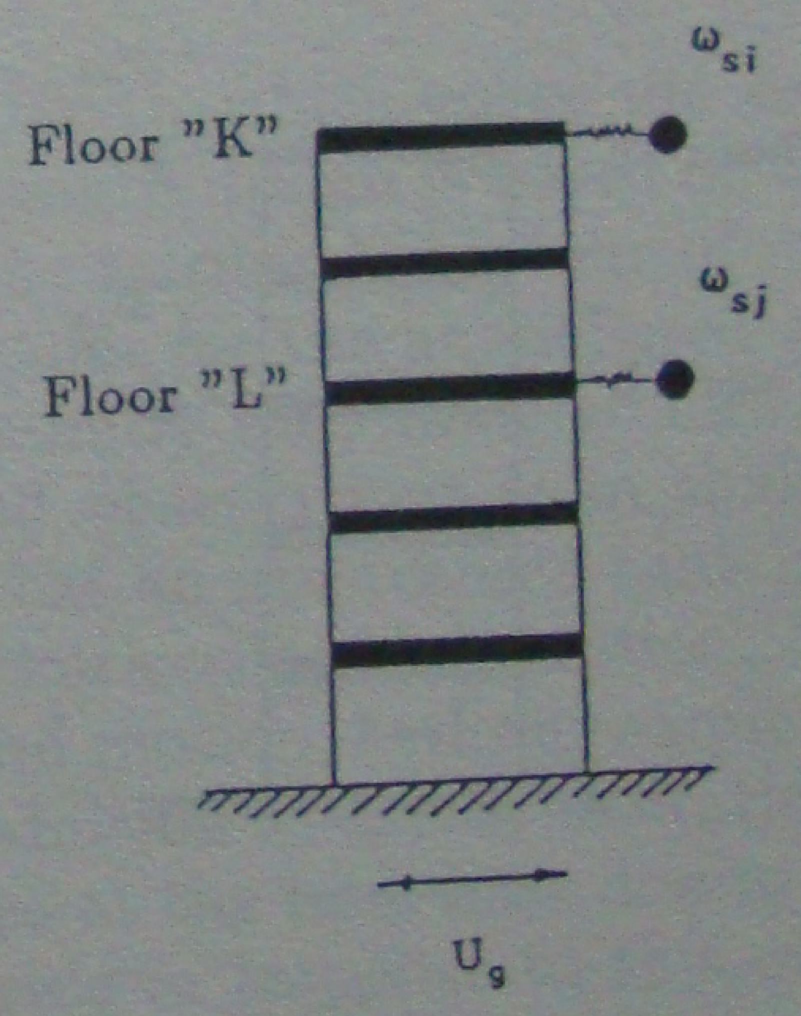


Figure 2. Primary-Double Oscillator System

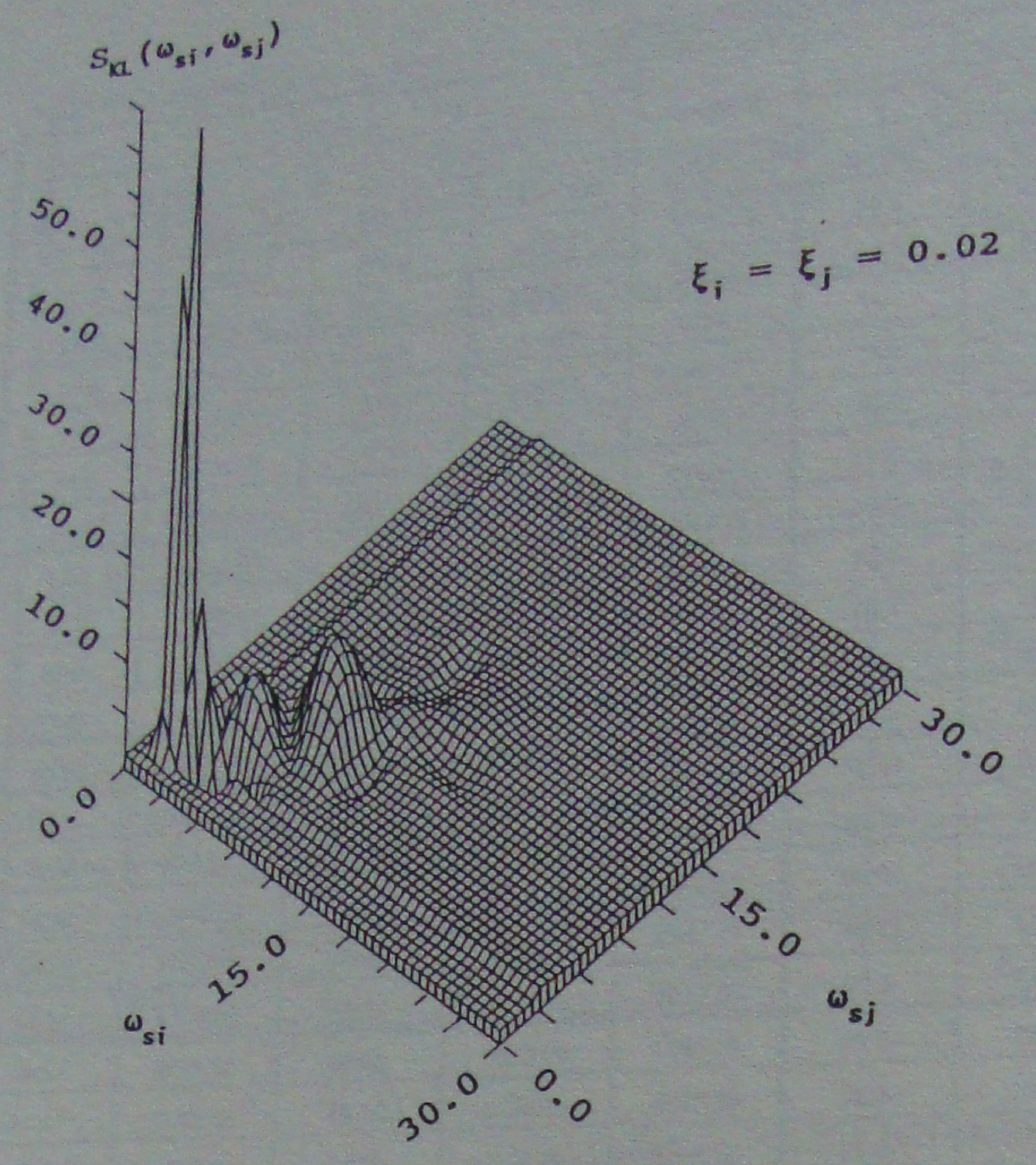


Figure 3. Cross Cross Floor Response Spectrum (CCFS)

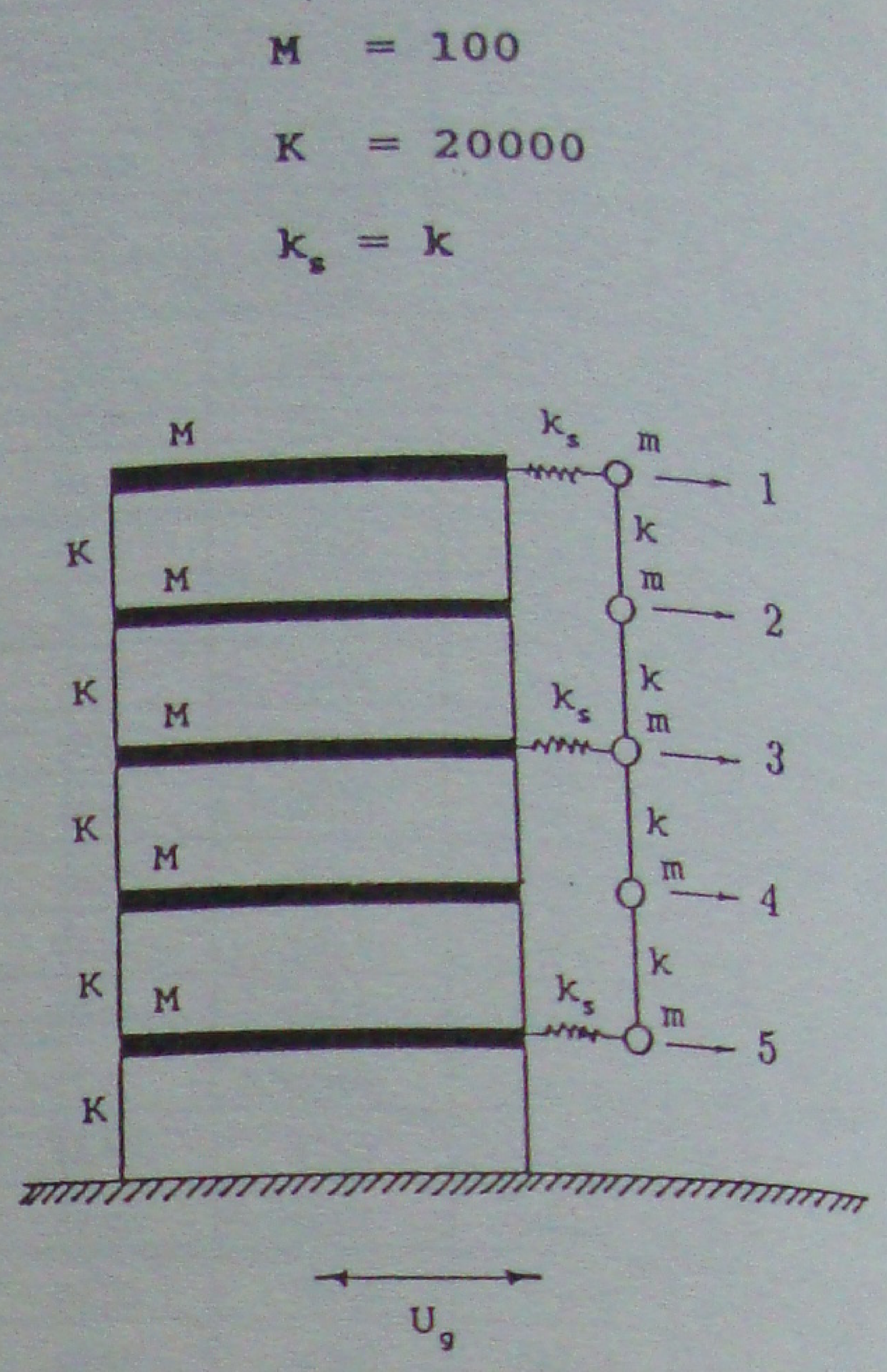


Figure 5. Model Considered in The Analysis

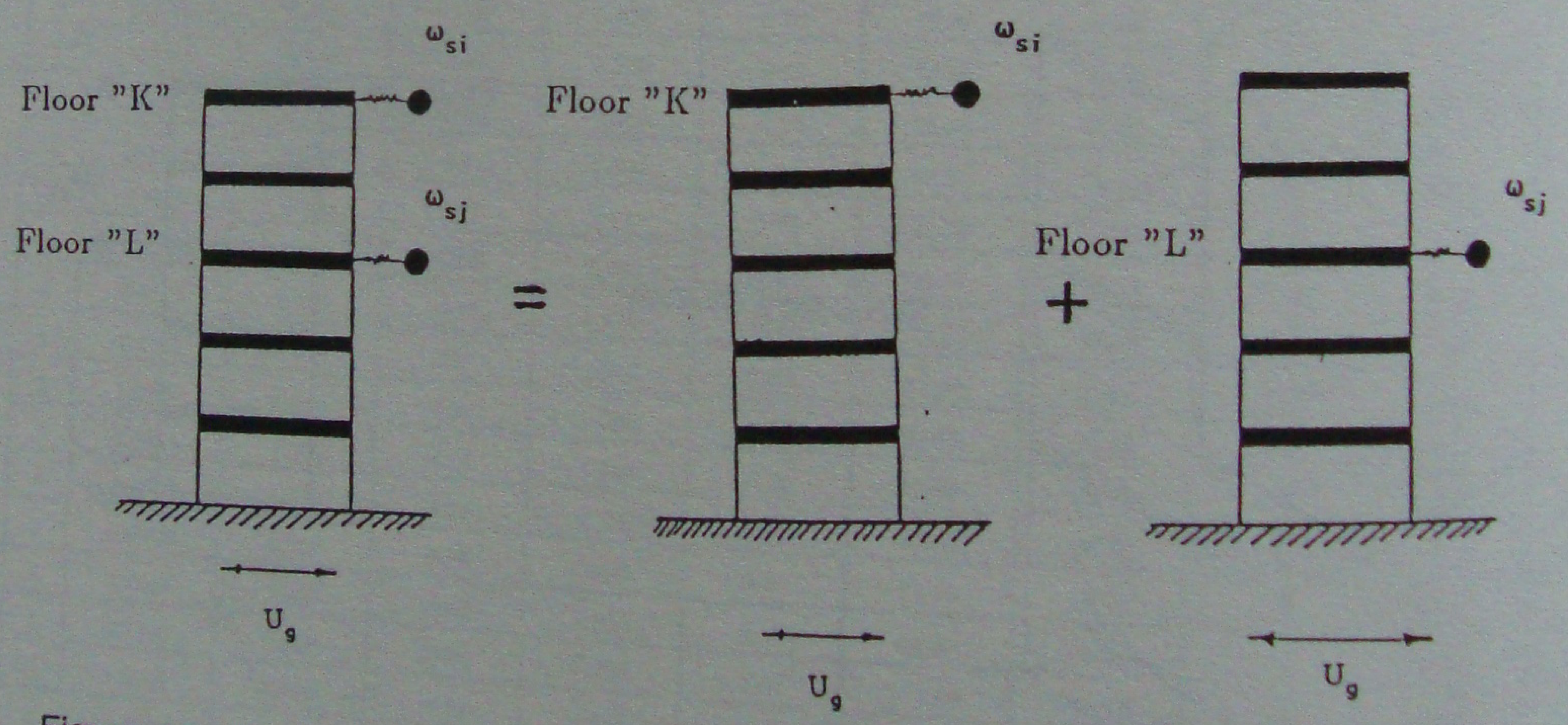


Figure 4. Replacement of The (N+2) DOF System with Two (N+1) DOF Systems